CALCULUS II EXAM 1 (FALL 2017) **NAME** ______________________________

1. Evaluate each definite integral.

   (a) (8 points) $\displaystyle\int_{\pi/4}^{\pi/3} \left(\frac{5}{x^2} + \sec^2 x\right) dx$

   (b) (8 points) $\displaystyle\int_{1}^{2} x\sqrt{x-1}\, dx$

   (c) (8 points) $\displaystyle\int_{0}^{1} \frac{4x+2}{x^2+x+5}\, dx$

   (d) (8 points) $\displaystyle\int_{0}^{\pi/4} \sin^3 2x \cos 2x\, dx$

2. (10 points) Sketch the curve $y = e^x - 1$ and
   (a) find the **signed** area between the curve and the interval $[-1, 2]$ on the x-axis;
   (b) find the **total** area between the curve and the interval $[-1, 2]$ on the x-axis.

3. (7 points) Sketch the region whose area is represented by the definite integral

$$\int_{-2}^{0} \sqrt{4 - x^2}\,dx,$$

   and evaluate the integral using an appropriate formula from **geometry**.

4. (8 points) Find the average value of the function $f(x) = \dfrac{1}{1 + 4x^2}$ over the interval $\left[0, \dfrac{1}{2}\right]$.

5. (3 points) If $\int_{-1}^{2} f(x)dx = 3$ and $\int_{2}^{5} f(x)dx = 7$, find $\int_{5}^{-1} f(x)dx$.

6. (12 points) Use $A = \lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ with x_k^* as the *right* endpoint of each subinterval to find the area under the curve $f(x) = x^3$ over the interval $[2, 4]$. (*No credit will be given for using other methods.*)
Formula: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$; $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$; $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$

7. (8 points) Let $F(x) = \int_3^x \sqrt{t^2+1}\, dt$.
   Find (a) $F(3)$ (b) $F'(2)$ (c) $F''(1)$

8. (8 points) Let $f(x) = \sqrt{x}$. Find all values of x^* in the interval $[0, 4]$ that satisfy the formula in the Mean Value Theorem for integrals.

9. (12 points) Sketch the region enclosed by the curves and find the area.

$$y^2 = x + 1, \quad y = x - 5.$$